

Classification of Future Electricity Market Prices

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Abstract—Forecasting short-term electricity market prices has been the focus of several studies in recent years. Although various approaches have been examined, achieving sufficiently low forecasting errors has not been always possible. However, certain applications, such as demand-side management, do not require exact values for future prices but utilize specific price thresholds as the basis for making short-term scheduling decisions. In this paper, classification of future electricity market prices with respect to pre-specified price thresholds is introduced. Two alternative models based on support vector machines are proposed in a multi-class, multi-step-ahead price classification context. Numerical results are provided for classifying prices in Ontario's and Alberta's markets.

Index Terms—Classification, demand-side management, forecasting, scheduling, smart grid, support vector machines.

I. INTRODUCTION

ELECTRICITY price is a key factor in determining short-term operating schedules and bidding strategies in competitive electricity markets [1]. Consequently, numerous data-driven approaches have been proposed for modeling and forecasting short-term electricity market prices [2]–[16]. The reported price forecasting errors generally range from approximately 5% to 36% and vary based on the technique used and the market analyzed. This range of error, however, is relatively high when compared to that of short-term electric load forecasting where errors usually range from 1% to 3% [17].

Various factors contribute to reduced accuracy of electricity price forecasting models; unpredictable forced outages [16], complex and changing price regimes [18], integration of intermittent energy sources [19], and implementation of reliability-based demand response programs [20] all introduce fluctuations and changes in electricity prices that may be extremely difficult to model accurately and consistently.

It is observed from the existing literature that traditional price forecasting models are generally developed for numerical prediction or point-forecasting. That is, existing models try to predict the exact value of prices at future hours by approximating the true underlying price formation process. However, not all market participants need to know the exact value of future prices in their decision-making process. For example, through the in-

troduction of “smart grid” technologies and new marketplace initiatives, it is expected that the demand-side interactions will be enabled to widely participate in electricity markets at the residential, commercial, and industrial levels [21]. Considering the on/off nature of most of electric loads, especially at the residential level, demand-side market participants may primarily react when prices exceed specific thresholds. Beyond these thresholds, the exact price of electricity would be considered unimportant since it is simply “too expensive”. Moreover, many demand-response products are designed having certain thresholds for electricity prices in mind [20], such as the hour-ahead dispatchable load program in the Ontario market [22]. Another example of threshold-based decision making can be found in electricity consumers with on-site generation facilities. These facilities only purchase electricity from the grid if the electricity market price are below the marginal cost of operating the on-site electricity generation equipment [12]. In these types of applications where the exact value of prices is not primarily required, the point-price forecasting problem can be reduced to price classification subproblems in which the class of future prices is of interest.

This paper proposes a short-term price classification method as an alternative to numerical price forecasting. In price classification, predictions are made with respect to whether the price is above or below pre-specified price thresholds defined by users based on their operation and planning objectives. Price classification is specifically useful when the exact value of future prices is not critically important. The main contribution of this paper is to propose a customized approach to predict the behavior of future prices where the specific forecasting needs of the users are taken into consideration.

The remainder of this paper is organized as follows: In Section II, a review of the background pertaining to this work is presented. The proposed models are discussed in Section III followed by the numerical results in Section IV. Finally, the main findings of this paper are summarized in Section V.

II. BACKGROUND REVIEW

In general, data-driven predictive models are built for either numerical prediction or classification. A numerical prediction model approximates the underlying process under consideration and is used to forecast future values for the variable of interest. Classification refers to the assignment of class labels to unlabeled data. This section presents a background review of literature pertaining to short-term electricity price forecasting, and discusses the steps of predictive model building.

A. Review of Short-Term Price Forecasting Literature

There are a wide variety of publications regarding numerical or point forecasting of future electricity prices; these works em-

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ploy a variety of different models and have had varying accuracy in their predictions. For example, the work presented in [4] describes neural networks-based models for forecasting prices in the Spanish and Pennsylvania, New Jersey, and Maryland (PJM) markets with an overall forecasting error of about 5%; however, the model accuracy was reported to collapse when only high price hours were concerned. Weighted nearest neighbors techniques are proposed in [5] and forecasting errors ranging between 5% and 16% were reported for the Spanish market; the variations in forecast accuracy over the studied period were attributed to various unpredictable factors including extreme weather conditions. A dynamic model based on system identification techniques is detailed in [6] with forecasting errors varying between 5% and 36% for the Italian, New England, and New York markets. Several statistical parametric and semi-parametric models are applied to forecasting in California and Nordic electricity market prices in [7] and errors ranging from 3% to 15% were reported. It was concluded in [7] that no single model could presently be chosen as the single best approach. A hybrid method, composed of support vector machines (SVMs) and a self-organized map, is applied to New England market prices in [8] with resulting forecasting errors of about 10% and 7% reported for the prices before and after the implementation of the standard market design, respectively. In [9] and [10], time series and neuro-fuzzy models are applied to forecasting Ontario's electricity prices; the forecast errors were reported to vary between 16% and 22% and the high forecast errors were attributed to the high volatility of prices in Ontario [9]. In [11], several linear and nonlinear models are employed to forecast Ontario prices for a three-year study period. The average forecasting errors in [11] vary between 22.98% and 31.86%, with a SVM-based model yielding the lowest errors. In another study [12] on forecasting prices in National Electricity Market of Australia, an SVM-based model is optimized using genetic algorithms and the resulting forecast errors are reported to vary between 16.39% and 23.26% for different time periods.

In addition to publications regarding numerical electricity price forecasting, several papers have included the estimation of prediction or confidence intervals. Among those, a method based on neural networks and Kalman filters is described in [13] and a different approach based on SVMs is detailed in [14].

Finally, [15] and [16] focus on the treatment and handling of price spikes and propose hybrid models to predict their occurrence.

B. Data-Driven Model Building

Building a data-driven predictive model has three main steps: data preprocessing, feature selection, and model selection. This section provides a literature review and discussion regarding these three steps. When reviewing each of the steps, both the pertinent background information as well as the methods employed in the present work are discussed.

1) *Data Preprocessing*: Data preprocessing focuses on the initial treatment of data and includes gathering of information on data statistics, anomalies, missing values, and necessary data transformations. In the context of modeling electricity market price data, the reported studies highlight two aspects applicable

to price data in this step. First is the problem of outliers, where prices do not follow the observed historical patterns [2]. Outliers or abnormal prices generally result from supply scarcity or unexpected operational events such as the forced outage of a generation unit. Manipulating the outliers and data smoothing has been reported [7]; however, it has also been argued that unusual prices in electricity markets reflect the reality of price volatility in these markets and thus should neither be removed nor manipulated [24].

Second, electricity prices are not stationary and show strong daily and weekly seasonalities [2], [5]. In order to achieve better stationarity in the data, several data transformation approaches such as differencing, Box-Cox, and wavelet transformations have been utilized [2], [7], [24]. However, stationarity is not always a necessary condition, depending on the underlying assumptions of the employed models; for example, time series models are limited to stationarity data, but not neural networks. In the present work, only data normalization is applied since it has been found to improve classification accuracy.

2) *Feature Selection*: In this step, a subset of features (i.e., inputs or explanatory variables) is chosen from an initial feature set that efficiently captures patterns in the data. Two major groups of feature selection techniques are filter and wrapper methods. In filter methods [25], features are assessed for their relevance in explaining the target variable and those with the highest relevance are selected. Filter methods are fast and simple but the potential disadvantage of them is that feature selection is isolated from the prediction model. In wrapper methods, the complete feature set is explored for a near-optimal subset and the relevance of features is evaluated by the accuracy of the final predictions. While wrapper methods have been shown to provide high prediction accuracy [25], they are computationally expensive when compared with filter methods.

In the context of forecasting electricity prices, the most popular features are historical price and load data. Other features such as day and hour indexes, transmission constraints, load levels of neighboring systems [9], [10], [26], variants of reserve margin [9], [16], generator outages and temperature [10], and availability of different types of generation resources [3], [4], [27] have also been reported with varying degrees of effectiveness [10], [26]. Filter methods are the most frequently reported feature selection technique where linear measures including cross-correlation and auto-correlation [6], [7], [9], [10], [26], [28] as well as nonlinear measures such as mutual information [4] are used to evaluate the relevance of candidate features to price.

In the present work, historical price, load, and reserve information are considered in the initial set of features. The support vector machine recursive feature elimination (SVMRFE) [29] and kernel-based feature vector selection (KFVS) [30] techniques are employed for feature selection. In SVMRFE [29], a ranked list of features is created, and through a forward search process, the subset of features with the highest impact on classification accuracy is selected. KFVS [30] maps feature vectors into a lower dimensional space and their effectiveness in improving classification accuracy is evaluated through a forward search method. Note that KFVS is presented in [29] as a filter method, but it is customized and used as a wrapper method in

the present work. These two methods result in reasonably accurate results at affordable computational costs. A comparative analysis of effectiveness of various feature selection algorithms for electricity market price classification is beyond the scope of this paper.

3) *Model Selection*: In the final step, a set of training instances are used to construct a classification model that describes the available data and can be used to label future observations. Classification models can be categorized into logic-based, perceptron-based, likelihood-based, and SVM-based approaches [31]. In logic-based models, prediction is performed by setting some logical rules that are learned from a training set. Perceptron-based models are driven on feed-forward neural networks in which the output is a function of the weighted sum of the inputs. In likelihood-based or statistical models, the prediction is performed by constructing a probability model based on the historical data.

In SVM-based models, the fundamental idea is to determine separating hyperplanes to distinguish different data classes in a way that the hyperplanes have the maximum possible distance from either of the data sets. SVM theory is explained in detail in [32]. A brief review of SVM modeling technique is also presented in the Appendix. SVMs can achieve accurate classification results using a small set of training instances compared to other classification techniques [32]. This feature is particularly important in electricity market price classification since regime changes have been observed over relatively short time periods [18]. Moreover, SVMs are relatively robust against outlier data in the training set due to the way they find the alignment of the hyperplanes [32]. This is important as outlier prices have been repeatedly observed in electricity market price patterns [15]. Support vector machines have previously been applied to several applications with competitive classification accuracy at reasonably low computational time [33]. Therefore, SVMs are employed as the core classifier in the present work.

In the present work, the forecast user considers $M+1$ pre-defined price thresholds T_1, T_2, \dots, T_{M+1} , where $p^{min} = T_1 < \dots < T_{M+1} = p^{max}$ and p^{min} and p^{max} are the minimum and maximum prices. In such case, there would exist M price classes, c_1, c_2, \dots, c_M , corresponding to $(p^{min} = T_1 \leq p_t < T_2), (T_2 \leq p_t < T_3), \dots, (T_M \leq p_t < T_{M+1} = p^{max})$ price ranges that are separated by $M-1$ classification boundaries. Thus, the price classification here is a multi-class problem.

SVM classifiers were originally developed for binary, two-class problems. To extend the binary SVM to multi-class problems, three alternative approaches are available [34]. In one-against-all approach, M independent classifiers are trained to classify the input data in an M -class problem. Each classifier decides whether or not a given training instance belongs to a certain class. The classifier whose decision function, $W \cdot \phi(X_i) + b$ in (9), has the highest value decides the class if the decisions of individual classifiers do not match. In one-against-one approach, an independent classifier is applied to each possible pair of classes for a given training instance in an M -class problem, and the class that gets the highest number of votes is chosen as the one to which that given candidate input data is assigned. Therefore, an M -class problem with $M-1$ decision boundaries requires $M(M-1)/2$ classifiers. Finally, in a single-machine

approach, all available classes are considered at the same time and one optimization problem is solved to classify the data into M classes.

A comparative study [34] has examined the three alternative multi-class approaches for a wide range of classification applications. While for a given set of data the three methods presented different levels of accuracy in some cases, none of the methods consistently outperformed others across all applications. In the present work, the three approaches were initially implemented and tested; however, the overall accuracy results were not found to be significantly different. Thus, the models presented in the following sections are based on the one-against-all approach.

III. PROPOSED SVM-BASED CLASSIFIERS

The present work focuses on 24-hour-ahead classification of hourly electricity market prices. It is assumed that previous prices up to hour t , denoted as $\{\dots, p_{t-1}, p_t\}$, are available and the objective is to determine price classes for the next 24 h. That is, the goal is to find classes of p_{t+k} for $k \in \{1, 2, \dots, 24\}$. For simplicity, it is also assumed that p_t is the price at the last hour of Day d and the 24 hourly price classes for Day $d+1$ are to be determined. Linear [2] and nonlinear [4] autocorrelation studies have shown that electricity market price time series are strongly autocorrelated. In other words, p_t has strong correlation with time lagged prices, for instance, $p_{t-1}, p_{t-2}, p_{t-3}, p_{t-24}, p_{t-48}, p_{t-72}, \dots$. Therefore, time lagged prices have been consistently included in the models proposed in the literature for numerical price forecasting [2], [4], [9].

If lagged prices are used as input to a model, however, all previous numerical values may not be available in the forecasting stage depending on the forecasting horizon. For example, consider forecasting p_{t+5} , that is 5 h in the future from the present time t . In this case, assume time lagged prices $p_{t-1}, p_{t-2}, p_{t-3}$ are among the model input features. However, values p_{t+4}, p_{t+3} , and p_{t+2} are unavailable as they are in the future from the present time t . In numerical price forecasting, an unavailable lagged price is normally replaced by its corresponding forecast value as the best guess. In the present work, however, the output of a classification model is no longer a numerical value but is a class, and thus, unavailable lagged prices cannot be replaced by their corresponding forecasts anymore. Keeping this issue in mind, two alternative classification models are proposed in the following sections. These proposed models do not have the unavailable lagged prices present in the feature sets. Also, exploratory simulations generating numerical forecasts for these values and including them into the feature set was not found to be a competitive alternative for the proposed models.

A. Model M1: Independent Classifiers for Each Hour

In this model, the price time series $\{\dots, p_{t-2}, p_{t-1}, p_t\}$ is broken into

$$\{\{\dots, p_{d-1}^{(h=1)}, p_d^{(h=1)}\}, \dots, \{\dots, p_{d-1}^{(h=24)}, p_d^{(h=24)}\}\}$$

subtime series, where $p_d^{(h=k)}$ represents price at Hour k on Day d . Under this model, the 24-hour-ahead classification problem is

broken into 24 one-step-ahead classification problems. To predict the class of $p_{d+1}^{(h=k)}$ using the data from D previous days, the set of initial features, denoted here by $X_{M1}^{(h=k,d+1)}$, can be written as

$$X_{M1}^{(h=k,d+1)} = \left\{ p_{d+1-\delta}^{(h=k)}, \Gamma_{M1}^{(h=k)}, \delta = 1, \dots, D \right\} \quad (1)$$

where $p_{d+1-\delta}^{(h=k)}$ is the price at Hour k on δ days previous to day $d+1$ and $\Gamma_{M1}^{(h=k)}$ represent the subset of all other non-price features such as load for Hour k for Model M1. Depending on the number of classes and the selected multi-class approach, each one-step-ahead classifier may have a different number of binary SVMs. This set of 24 independent one-step-ahead classifiers is referred to as Model M1.

The disadvantage of Model M1 is that neither the price autocorrelation information nor the most recent price information is represented in the model. The SVMRFE approach [29] is used to select the final set of features out of $X_{M1}^{(h=k,d+1)}$ for each of the 24 one-step-ahead classifiers.

B. Model M2: Independent Classifiers for Each Hour Considering Price Autocorrelations

This model is similar to Model M1 in that price at each hour is classified independently. However, unlike Model M1, all 24 hourly prices of D previous days are considered rather than only the historical prices for Hour k . Thus, the initial feature set for predicting the class of $p_{d+1}^{(h=k)}$ can be written as the following matrix form:

$$X_{M2}^{(h=k,d+1)} = \begin{bmatrix} p_d^{(h=1)} & p_{d-1}^{(h=1)} & \dots & p_{d+1-D}^{(h=1)} \\ p_d^{(h=2)} & p_{d-1}^{(h=2)} & \dots & p_{d+1-D}^{(h=2)} \\ \vdots & \vdots & \ddots & \vdots \\ p_d^{(h=24)} & p_{d-1}^{(h=24)} & \dots & p_{d+1-D}^{(h=24)} \\ \Gamma_{d,M2}^{(h=k)} & \Gamma_{d-1,M2}^{(h=k)} & \dots & \Gamma_{d+1-D,M2}^{(h=k)} \end{bmatrix} \quad (2)$$

where $\Gamma_{d+1-\delta,M2}^{(h=k)}$, $\delta \in \{1, \dots, D\}$, represents the subset of all other non-price features from Day $d+1-\delta$ considered for Hour k . Observe that all of the 24 hourly prices from D previous days are used as features for each individual hour, that is, for $k = 1, 2, \dots, 24$. The difference in feature sets for different hours in this model lies in $\Gamma_{d+1-\delta,M2}^{(h=k)}$. Although the unavailable price information is not represented in this model either, the historical price autocorrelation information is represented in the initial set of features as the price patterns in the previous days are retained. Similar to Model M1, the number of binary SVMs in this model also depends on the number of classes and the selected multi-class approach. The set of 24 hourly classifiers is referred to as Model M2.

Note that for Model M2 the initial feature set is a “matrix”, as opposed to the “vector” of features for Model M1. Thus, the task in feature selection stage for Model M2 is to select the most informative days from the D previous days, i.e., the most informative columns of (2). This retains intra-day price and demand patterns and ensures that if a certain day is found informative, its full price pattern is represented in the model. Therefore, feature selection in this case is in fact “feature vector” selection, and hence, the KFVS approach of [30] is used for this model.

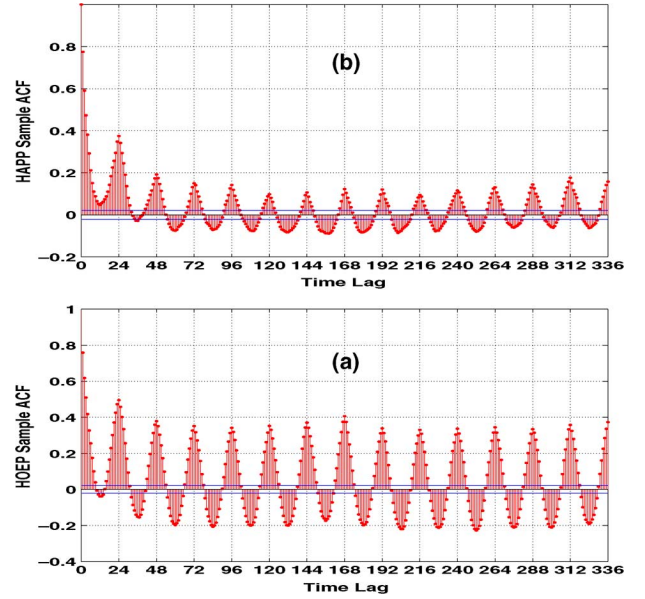


Fig. 1. Sample ACF for (a) the HOEPs and (b) the HAPPs for year 2008.

IV. NUMERICAL RESULTS

Historical data from the Ontario and Alberta electricity markets, which have the some of most volatile prices in North America, are selected for numerical simulations in this work. Ontario’s physical electricity market is a real-time joint energy and reserve market and is cleared every 5 min. The hourly average of cleared energy prices is referred to as Hourly Ontario Energy Price (HOEP) and applies to most demand- and supply-side wholesale market participants. Numerical forecasting errors ranging from 16% to 22% have been reported in the literature for the HOEP [9], [10]. Alberta’s market is an energy-only, real-time market which is cleared every minute and the average of the 60 market clearing prices over an hour, referred to as the Hourly Alberta Pool Price (HAPP), is used as the basis of financial settlements.

Fig. 1 depicts the sample autocorrelation functions (ACF) of the HOEP and HAPP time series for year 2008. Strong autocorrelations with the lagged prices, especially the first few lags and the seasonal daily and weekly lagged prices, are evident for both time series. Also observe that autocorrelations are stronger for the HOEPs compared to those of the HAPPs.

For Ontario’s market, price, load, and reserve information are considered in the initial set of features. For representing system reserve margin, the predicted supply cushion (PSC) is considered and is defined as $PSC = ((PAS) - (FD + RR)) / (FD + RR)$, where PAS is the predicted available supply, FD is the forecast demand, and RR is the reserve requirement. For Alberta’s market, price and load information are considered in the initial set of features. There is no reserve metric information included in the Alberta studies since this information is handled in an independent auction and some of the data is not publicly available.

Four classification thresholds are considered for each of the markets: $T_1^{HOEP} = -2000$, $T_2^{HOEP} = 50$, $T_3^{HOEP} = 100$, and

$T_4^{\text{HOEP}} = 2000$ for Ontario and $T_1^{\text{HAPP}} = 0$, $T_2^{\text{HAPP}} = 90$, $T_3^{\text{HAPP}} = 180$, and $T_4^{\text{HAPP}} = 1000$ for Alberta, with all values in \$/MWh. $T_1^{(\cdot)}$ and $T_4^{(\cdot)}$ are, respectively, the price floor and price cap defined by the applicable market rules in Ontario and Alberta. The values of $T_2^{(\cdot)}$ and $T_3^{(\cdot)}$ are, respectively, selected as the annual average and twice the annual average of the prices for year 2008. In practice, however, the users would normally define the thresholds based on their own operating criteria. For the above price thresholds, the price data are classified into three classes, that is, c_1 for prices between T_1 and T_2 , c_2 for prices between T_2 and T_3 , and c_3 for prices between T_3 and T_4 .

The standard binary SVM as discussed in the Appendix is used as the core classifier for this research. Since Models M1 and M2 classify hourly prices independently, 24 independent binary SVMs would be required for each model for a two-class problem. Considering the three assumed classes and the one-against-all multi-class approach, each of the models M1 and M2 are therefore composed of 72 independent binary SVMs.

Two popular kernel functions, radial basis and polynomial, are examined to map the data into a high-dimensional space. The Gaussian radial basis function, $K(x_i, x_j) = \exp(-(\|x_i - x_j\|^2 / 2\zeta^2))$, was found to yield higher overall accuracy than the polynomial approach. The numerical value of ζ was decided by trial and error. For a given day, we started with the variance of normalized prices over the previous 35 days and built the classification model. We also built the models with other values of ζ , i.e., from $\zeta = 0.8 \times \text{variance}$ to $\zeta = 1.2 \times \text{variance}$, with 1% increments. We repeated this process for several randomly selected days over the study period of year 2008. It was observed that the value of ζ equal to, or very close to, the 35-day variance generally resulted in highest classification accuracies for the examined days. Thus, the 35-day moving variance of the normalized prices was used for the value of ζ to build the daily classification models. In addition, we observed that the penalty term γ in the SVM objective [see (8) in the Appendix] did not contribute to the overall model accuracy and thus we set it to zero in the final tested models.

The proposed models are built and tested for each and every day of year 2008. For each given day, historical data from the 35 previous days are considered in the initial set of features, that is, $D = 35$. The choice of 35 days was based on trial and error and needs to be reexamined if the models are applied to other market prices. The SVM classifiers and the feature selection algorithms are implemented in MATLAB, and processing was performed on a single core commodity desktop system. To meet the practical data availability time-lines in real-life markets, the computational time was limited to 50 min and was considered as a stopping criterion when searching the feature space by the feature selection algorithms. In addition, it is anticipated that multithreading, multicore processing systems, and vectorization of the algorithms would likely improve the computational effectiveness considerably.

The mean percentage classification error (MPCE) is used as the overall measure of classification error in this paper. MPCE is defined as: $\text{MPCE}(\%) = 100 \times (N_{mc}/N_{tot})$, where N_{mc} is

the number of misclassifications and N_{tot} is the total number of classification instances. Given the number of models investigated and the consideration of three classes in this work, presenting other more detailed error/accuracy measures, such as a confusion matrix, is not possible due to page limitations.

A. Model M1

Considering the structure proposed in Section III-A, Model M1 is trained and tested for classifying the HOEPs and the HAPPs using three alternative initial feature sets. The alternative initial feature sets are explored to examine whether additional information can contribute to improved model accuracy in conjunction to historical pricing. In the first scenario, Model M1 is trained using an initial feature set which only consists of historical prices. Thus, in this case, the set of initial features for classifying $p_{d+1}^{(h=k)}$, is considered as follows:

$$X_{M1,P}^{(h=k,d+1)} = \{p_{d+1-\delta}^{(h=k)}, \delta = 1, \dots, 35\}. \quad (3)$$

In other words, prices at Hour k for 35 previous days are considered as inputs to the model. Twenty-four three-class classifiers are built for $k = 1, \dots, 24$, and the resulting 24-hour-ahead classification model is referred to here as $M1_{(P)}$.

In the second scenario, load data are also added to the set of initial features, as follows:

$$X_{M1,P,L}^{(h=k,d+1)} = \{X_{M1,P}^{(h=k,d+1)}, L_{d+1-\delta}^{(h=k)}, \hat{L}_{d+1}^{(h=k)}\} \quad (4)$$

where $\delta \in \{1, \dots, 35\}$, $L_{d+1-\delta}^{(h=k)}$ represents the historical load for Hour k of Day $d + 1 - \delta$ and $\hat{L}_{d+1}^{(h=k)}$ is the load forecast for Hour k of Day $d + 1$. Note that actual demand values are used in the simulations as historical demand data, and load forecasts made available by the Ontario and Alberta market operators are used for $\hat{L}_{d+1}^{(h=k)}$. The resulting 24-hour-ahead classification model is similarly denoted here by $M1_{(P,L)}$.

A third scenario is applied to the Ontario market only and includes PSC data for 35 previous days, i.e., $\text{PSC}_{d+1-\delta}^{(h=k)}, \delta \in \{1, \dots, 35\}$, as well as PSC for the target day, i.e., $\text{PSC}_{d+1}^{(h=k)}$, into the initial feature set as follows:

$$X_{M1,P,L,\text{PSC}}^{(h=k,d+1)} = \{X_{M1,P,L}^{(h=k,d+1)}, \text{PSC}_{d+1-\delta}^{(h=k)}, \text{PSC}_{d+1}^{(h=k)}\}. \quad (5)$$

Note that the PSC values used in the simulations are all based on predicted values of demand and supply in both model building and forecasting stages. The resulting 24-hour-ahead classification model is referred to here by $M1_{(P,L,\text{PSC})}$.

The classification errors of the three scenarios for Model M1 are presented in Table I.

B. Model M2

Model M2 is also trained for classification of both market price time series in three different scenarios. The first scenario considers only historical prices in the initial feature sets

TABLE I
MONTHLY MPCES (%) FOR ALL MODELS

	Ontario								Alberta				
	M1(P)	M1(P,L)	M1(P,L,PSC)	M2(P)	M2(P,L)	M2(P,L,PSC)	MA	SDNN	M1(P)	M1(P,L)	M2(P)	M2(P,L)	MA
January	15.5	15.2	15.1	12.2	12.1	11.7	20.2	25.2	23.1	21.6	12.9	12.4	29.4
February	15.5	16.4	15.8	11.8	11.5	11.2	39.4	40.3	23.4	22.8	11.9	11.2	24.9
March	12.9	12.8	12.4	9.1	9.0	9.1	39.9	33.4	20.4	21.0	10.4	9.8	28.1
April	12.8	12.5	12.5	7.2	6.9	6.7	29.2	33.4	20.1	20.1	10.0	9.4	54.3
May	11.7	11.2	11.0	6.5	6.3	6.2	23.0	32.3	18.2	18.3	9.1	8.5	35.3
June	12.6	12.8	12.6	6.9	6.7	6.5	36.1	23.7	19.0	19.2	8.8	8.9	36.3
July	13.0	12.9	12.8	7.1	7.3	7.5	36.3	29.8	17.9	18.4	8.2	8.5	27.7
August	12.5	12.4	12.2	7.0	7.1	6.9	31.9	32.7	16.9	17.7	9.0	8.2	27.8
September	13.3	13.2	13.3	7.9	7.8	7.2	31.3	28.7	19.2	19.7	10.1	9.0	33.3
October	15.7	15.5	14.9	7.8	8.1	8.2	35.4	33.3	20.6	20.6	11.7	11.6	41.0
November	17.8	17.9	17.5	11.4	11.5	11.9	35.6	37.6	22.9	22.4	13.5	13.3	29.0
December	17.7	18.1	17.7	12.5	12.4	12.8	26.9	34.1	23.0	21.9	13.7	13.6	24.9
Average	14.3	14.2	14.0	9.0	8.9	8.8	31.9	31.6	20.4	20.3	10.8	10.4	32.5

and these feature sets are identical for all the 24 h, i.e. in (2) $\Gamma_{d+1-\delta, M2}^{(h=k)} = \emptyset$, $\delta \in \{1, \dots, 35\}$. In this scenario, the initial feature matrix is composed of 35 columns, each column consisting the 24 hourly prices of a historical day.

Feature vector selection in this scenario is conducted according to the KFVS algorithm in [30] which requires a mapping function to reduce the dimension of the original feature vectors. The KFVS algorithm does not impose any limitations on the characteristics of the mapping functions, except that they must have a lower dimension than the original vector. This study considers the general features of electricity prices, such as high daily price volatility and price spikes, and a mapping function composed of daily geometric mean of hourly prices, intra-day price volatility, and daily average of hourly prices is used. Intra-day price volatility for Day d is defined as $\sigma_d = \sqrt{\sum_{k=2}^{24} (r_{k,d} - \bar{r}_d)^2} / 23$, and is a measure of hourly price variations where $r_{k,d} = \ln(p_d^{(h=k)} / p_d^{(h=k-1)})$ is the logarithmic price return and \bar{r}_d is $(1/23) \sum_{k=2}^{24} r_{k,d}$ [35]. Use of the geometric mean mitigates the negative impact of extreme outlier prices whereas the price volatility ensures that intra-day price fluctuations are represented. This mapping function effectively maps the columns of the initial feature matrix from a 24-dimensional space into a three-dimensional space. The outcome of the feature selection process here is a set of previous days, out of the 35 initially considered days, whose data describe price classes the best. Note that when certain days are selected by the KFVS algorithm as informative days, all of the 24 hourly prices of the selected days are included in the feature set of the SVM classifiers.

In the second scenario, load data are added to the set of initial features. Thus, the non-price feature sets can be written as follows:

$$\Gamma_{d+1-\delta, M2, L}^{(h=k)} = \{L_{d+1-\delta}^{(h=1)}, \dots, L_{d+1-\delta}^{(h=24)}, \hat{L}_{d+1}^{(h=k)}\} \quad (6)$$

where $\delta \in \{1, \dots, 35\}$. Observe that the historical load for all the 24 h of the previous days are included in the feature sets and the load forecast is the only feature that varies in the features

sets for different hours. For feature selection, daily average demand, intra-day demand volatility, and $\hat{L}_{d+1}^{(h=k)}$ are considered as the mapped features in addition to those considered in the first scenario. Using intra-day demand volatility ensures that the daily demand fluctuations are represented in the feature selection process.

In the third scenario, and for Ontario's market only, the PSC data are added to the set of initial features as follows:

$$\Gamma_{d+1-\delta, M2, L, PSC}^{(h=k)} = \left\{ \Gamma_{d+1-\delta, M2, L}^{(h=k)}, PSC_{d+1-\delta}^{(h=1)}, \dots, PSC_{d+1-\delta}^{(h=24)}, PSC_{d+1}^{(h=k)}, \delta = 1, \dots, 35 \right\}. \quad (7)$$

In this scenario, daily average PSC, daily geometric mean PSC, and $PSC_{d+1}^{(h=k)}$ are added to the mapped features of the second scenario. The resulting 24-hour-ahead models in the three scenarios are referred to here by M2(P), M2(P,L), and M2(P,L,PSC). The classification errors for Model M2 in the three scenarios are also presented in Table I. Observe that Model M2 significantly outperforms Model M1 for both markets.

The results presented in Table I indicate that additional load and reserve information has not significantly improved classifications accuracy in the models. In order to explain this observation, the fluctuations of price and load in year 2008 for Alberta's system are presented in Fig. 2. In this figure, the load-price pairs are sorted in a descending order with respect to load values. Although very high prices are more likely at high demand hours, moderate and high prices often occur over a wide range of load. For example, class c_3 in Alberta studies includes prices over \$180/MWh and corresponding demand could vary between about 7000 to 9000 MW. Thus, the additional information carried by the demand data has a marginal value in identifying price classes. Similar behavior was observed for load and PSC in Ontario's market. Additional load and reserve information, however, has been useful in previously published numerical price forecasting studies [3], [4], [9].

Also observe from the results presented in Table I that the classification accuracy for the cold months of November to February are lower compared to other periods. Both Ontario and Alberta markets experience high demand during cold months

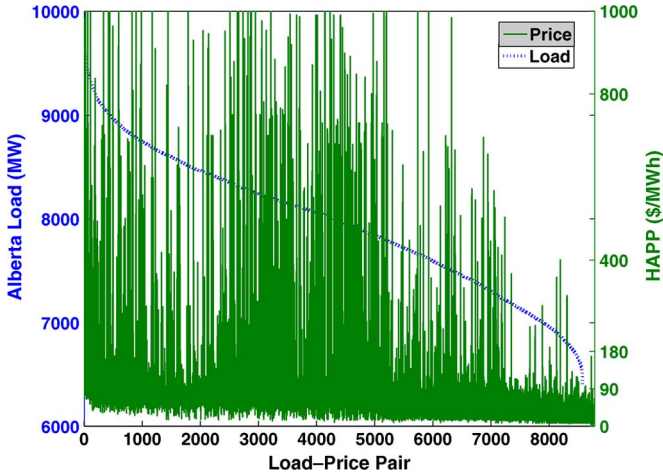


Fig. 2. Price-demand fluctuations for Alberta's market in year 2008.

which results in highly volatile prices, and thus, more difficult to predict [36].

C. Performance Comparison

Classification accuracy of the models proposed in this paper is compared to that of the numerical forecasts presented in [9]. Numerical HOEP forecasts are presented in [9] for a 42-day test period, consisting of three two-week periods in spring, summer, and winter 2004. The forecasts are generated using ARIMA, transfer function (TF), and dynamic regression (DR) models. The pre-dispatch price (PDP) forecasts published by the Ontario Independent Electricity System Operator for the same test period are also considered. These price forecasts are classified according to the thresholds specified in the present work, and the resulting 42-day MPCEs are found to be 21.52%, 17.85%, 17.26%, and 26.39% for the ARIMA, TF, DR models, and PDPs, respectively. The ARIMA model and the TF and DR models were trained using 28 and 70 days of historical data, respectively, which were decided by trial and error. Also note that these models were separately estimated for each and every day of the 42-day period. The proposed models in the present paper were also trained using the same set of features from year 2004, but based on 35 historical days. Models M1 and M2 of the present work generally outperformed those in [9] in all scenarios, with Model M2_(P) having the lowest 42-day MPCE of 6.84%.

In addition, we built a moving average (MA) model by assigning the average of prices over a seven-day moving window as the price forecasts of the next day, for each and every day of year 2008. The forecasts of the MA model are also classified according to the thresholds defined for the HOEP and the HAP, and the resulting MPCEs are presented in Table I. Observe that the classification accuracy of the proposed models is significantly higher than that of the MA models for the two price sets.

We also generated numerical forecasts for Ontario market prices of year 2008 using the model of [37] which is based on similar day method and neural networks (SDNN). The SDNN

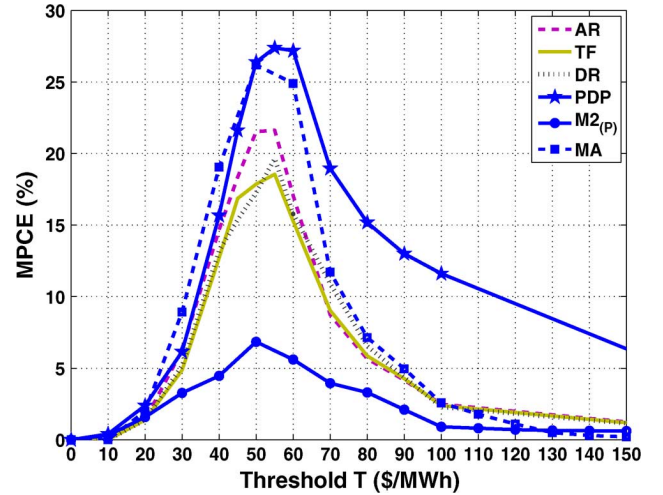


Fig. 3. Sensitivity of the MPCE to threshold T , the HOEP.

models were trained using the same set of information as for model M2_(P,L) in Section IV-B. Next-day forecasts were generated for each and every day of year 2008, and the results were classified according to the same selected threshold values for Ontario, as discussed earlier. Note that the SDNN models were individually retrained for forecasting prices of each specific day. The monthly average errors are presented in Table I under column SDNN. The results show that classification accuracy of our models is significantly better than that of the SDNN model.

D. Sensitivity of the Results to the Selected Thresholds

To investigate the sensitivity of classification errors to price thresholds, we considered a two-class version of Model M2_(P) where the classes c_1 and c_2 referred to prices under and above a single threshold T . The model was trained for classifying the HOEP using different values of T ranging from 0 to 150 \$/MWh in \$10/MWh increments. To make a sensitivity comparison, the forecasts of the ARIMA, TF, and DR models of [9] and the PDPs, are also classified according to T . Forecasts of the MA model for the 42-day period were also calculated and classified according to T . Note that the results of Model M2_(P) are also from the same 42-day test period in year 2004 as in [9]. The resulting classification errors for each threshold T are presented in Fig. 3.

Observe from Fig. 3 that the highest classification errors occur when the threshold is around \$50/MWh, which is the year 2004 annual average of the HOEP. In order to explain this sensitivity, consider the extreme case in which the threshold is very small, say \$1/MWh. The 2004 historical HOEPs were all above \$5/MWh, and a simple model can capture this trend and predict the future prices to be above \$1/MWh with a high accuracy. This would lead to a very small classification error. In another extreme case, assume a very large threshold, say $T = \$2000/\text{MWh}$. Considering the historical HOEP behavior, it is not difficult to predict that prices are under this threshold most of the time, which would also result in a very small classification error. Predicting the prices with respect to any

threshold in between the two extreme cases will be more complicated, depending on how often prices fall above or under that threshold. HOEP values fall below $T = \$50/\text{MWh}$ for about 60% of hours in 2004, indicating that prices are distributed around this threshold almost evenly and, thus, are harder to classify. Note also from Fig. 3 that similar sensitivities to the threshold value can be observed for the numerical forecasts of the ARIMA, TF, and DR models and the PDPs. Similar sensitivities were observed for the HAPP, but details are not discussed here.

From the results presented in this section and Section IV-C, price classification may also have relatively high errors, especially if the threshold is defined around the mean price. However, these errors are significantly lower compared to numerical price forecasting. The cost of this lower error is the loss of exact price information, which may not be important in specific applications. For example, an industrial consumer may decide to shut down a production line if prices hit a certain threshold.

V. CONCLUSIONS

Classification of future electricity prices with respect to a number of pre-specified price thresholds is discussed in this paper. Multi-class SVMs are employed as the core classifier for multi-step-ahead classification. Two different models are proposed and analyzed considering different sets of input pricing data, and each of these models is further evaluated considering combinations of price, load, and predicted supply cushion. The models are tested using data from both the Ontario and Alberta electricity systems, and both of these markets exhibit high price volatilities. Where possible, classification accuracy of these models is compared against the data available in previous literature.

The simulation results demonstrate that the proposed price classification models provide significantly more accurate results compared to the available numerical price forecasting models that are available for comparison. This is particularly important in markets with high price volatility where numerical price forecasting is more difficult and classification of these predicted prices leads to incorrect analysis. The cost associated with this higher classification accuracy is the loss of exact price value information; however, market participants whose operation decisions are based on certain price thresholds can realize an improvement in their operating strategies.

APPENDIX

A binary SVM is a “maximum margin classifier” that separates the training data into two classes while maximizing the margin between the two [32]. Assume $\{(X_i, Y_i), X_i \in \mathbb{R}^N, Y_i \in \{-1, 1\}, i \in \{1, \dots, I\}\}$ is the set of I linearly-separable training instances, where X_i is the N -dimensional vector of features and Y_i represents the class of instance i , say $Y_i = 1$ for one class and $Y_i = -1$ for the other. The goal of training the SVM is to find two maximum-margin hyper-planes $W \cdot X_i + b = 1$ and $W \cdot X_i + b = -1$ which would separate the instances having $Y_i = 1$ from those having $Y_i = -1$. It can be shown that $2/\|W\|$ is the margin bounded by the two

parallel hyper-planes. To maximize this margin, and to avoid non-convexity, the following equivalent quadratic optimization problem is solved to find SVM parameters W and b :

$$\min_{W, b} \frac{1}{2} \|W\|^2 + \gamma \sum_{i=1}^K \epsilon_i \quad (8)$$

$$\text{s.t. : } Y_i (W \cdot \phi(X_i) + b) \geq 1 - \epsilon_i \quad i \in 1, \dots, I \quad (9)$$

where $\phi(X_i)$ maps the input vector X_i into a high-dimensional space, γ is a penalty factor, and the ϵ_i terms are slack variables. The mapped input feature and the penalty term are used when the data are linearly nonseparable. Constraint (9) discourages any instance from crossing over the hyper-planes and falling into the margin, and in case one does, this constraint penalizes the objective function accordingly. The optimization problem in (8) and (9) is solved using the original optimal hyper-plane algorithm proposed by Vapnik. However, other approaches, such as sequential minimal optimization, are also available.

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